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Natural sway frequencies and damping ratios of trees: concepts, review and synthesis of previous studies

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Abstract Previous studies that measured the natural frequencies and damping ratios of conifer trees were reviewed and results synthesized. Analysis of natural frequency measurements from 602 trees, belonging to eight different species, showed that natural frequency was strongly and linearly related to the ratio of diameter at breast height to total tree height squared (i.e., DBH/H^2). After accounting for their size, pines (*Pinus* spp.) were found to have a significantly lower natural frequency than both spruce (*Picea* spp.) and Douglas-fir (*Pseudotsuga* spp.). Natural sway frequencies of de-branched trees were significantly higher than those of the same trees with the branches intact, and the difference increased with an increasing ratio of DBH/H^2 . Damping mechanisms were discussed and methods for measuring damping ratio were presented. Analysis of available data suggested that internal damping ratios were typically less than 0.05 and were not related to tree diameter. External damping was mainly due to aerodynamic drag on the foliage and contact between the crowns of adjacent trees. Analysis of data from previous wind-tunnel studies indicated that damping due to aerodynamic drag is a nonlinear function of velocity. Damping due to crown contact has been suggested by a previous author to be a function of both the distance to and the size of adjacent trees. Therefore, in uniformly spaced stands it may be possible to model crown contact damping as a function of stand density

index (SDI), a common forestry measure which incorporates both of these variables.

Keywords Wind damage · Tree mechanics · Natural frequency · Damping ratio

Introduction

Wind is an important natural disturbance agent in forests in many regions of the world as it influences stand structure and development (e.g., Nowacki and Kramer 1998). However, where forests are managed for wood production, damage from wind storms is viewed as a problem (Quine and Gardiner 1991; Everham 1995). In response to this damage, a number of authors have developed classification schemes to address the risk of wind damage (Quine 1995). Many of these schemes have a strong empirical basis and provide only a qualitative assessment of risk (i.e., low, medium, high etc.). More quantitative assessments of risk, and the investigation of the effects of both individual tree characteristics and stand structure on this risk, can be achieved using mechanistic models which attempt to model the processes that cause wind damage. A major component of these mechanistic models is a sub-model which predicts the response of a tree to a particular wind load. The simplest models treat the applied wind load acting on a tree as a static force (i.e., invariant with respect to time) and use engineering beam theory to determine the deflection from the vertical, the maximum bending moment at the base of the tree, as well as the vertical distribution of stresses within the tree. Although understanding the static behavior of trees provides a good basis for understanding their overall behavior, it is a simplification of reality. Trees are dynamic systems and their behavior varies with time. The response of a tree is frequency dependent with the tree responding most to wind gusts at frequencies close to its resonant frequency and its harmonics (Gardiner 1992). In these situations the dynamic effects are likely to increase the bending of stems and hence the load on the root system (Milne 1991).

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There are two approaches to quantifying the response of a tree to a given fluctuating wind force. The first requires both the wind force and tree response spectra to be measured and a mechanical transfer function developed to relate the two (e.g. Holbo et al. 1980; Mayer 1987). Alternatively, if we have information on the dynamic properties (i.e., natural frequency and damping) of trees then it is possible to characterize their response to any known applied force by employing a mechanical model (e.g. Baker 1995; Kerzenmacher and Gardiner 1998; Flesch and Wilson 1999). This latter approach allows the effects of manipulating various tree and stand characteristics to be investigated. However, information on natural frequencies and damping ratios of trees is sparse and is often contained in unpublished reports. This paper summarizes available information on these properties, reviews studies that have attempted to predict them from certain tree characteristics, and draws comparisons between species.

Natural frequency

Periodic motions can be described by a few basic characteristics, the most important of which are amplitude (y); period (T) and frequency (f). For simple harmonic motion, amplitude is defined as the maximum displacement from the rest position. Period is the time required to complete one cycle of motion. Frequency is the inverse of period, i.e., the number of repetitive cycles which occur during a unit time:

$$f = 1/T \quad (1)$$

The natural frequencies of a tree are the frequencies it will inherently oscillate at under free vibration, and at which resonance will occur if it is excited at one or more of these frequencies. These different natural frequencies are associated with the various modes of vibration that can occur in the tree. The mode of vibration is determined by the number of half sine waves that occur in the vibrating body, and higher modes are characterized by higher natural frequencies. The first natural frequency normally dominates the response of a tree and most studies have focused on determining this property.

Previous studies

Estimates of natural frequency have been obtained from studies where trees were forced to sway with an attached rope and then released and allowed to adopt their natural frequency (i.e., damped free vibration). One of the earliest known studies was conducted by Sugden (1962) who used a stopwatch to measure the natural sway periods of 826 red (*Pinus resinosa* Ait.) and white pine (*Pinus strobus* L.) trees growing in four plantations and one native stand at the Petawawa Forest Experiment Station, Ontario, Canada (Table 1). During the 1960s, the British Forestry Commission used the same method to measure the sway periods of 143 plantation-grown trees from five different

species: Sitka spruce (*Picea sitchensis* Bong Carr), Douglas-fir (*Pseudotsuga menziesii* Mirb. Franco), Norway spruce (*Picea abies* L.), Lodgepole pine (*Pinus contorta* Dougl. ex Loud.) and Corsican pine (*Pinus nigra* Arnold). These data were presented and analyzed by Mayhead (1973a). A further study was performed by the British Forestry Commission in the 1970s (Mayhead et al. 1975). This time the sway periods were calculated from displacement measurements made using a chart recorder. More recent studies (Milne 1991; Gardiner 1992; Roodbaraky et al. 1994; Flesch and Wilson 1999) have used portable dataloggers to record the output from tree displacement sensors. These sensors ranged from rotary or linear potentiometers connected to the tree by thin wires (Milne 1991; Gardiner 1992; Roodbaraky 1994), to bi-axial tilt sensors (Flesch and Wilson 1999) and prism-based systems (Hassinen et al. 1998).

Accelerometers have also been used to measure the motion of trees (White et al. 1976; Peltola et al. 1993). However, the initial position of the tree must be estimated, and any error is then compounded during the two stages of integration required to obtain displacement. This leads to an accumulating error in calculated displacement, also known as "zero drift" (White et al. 1976; Peltola et al. 1993; Hassinen et al. 1998). The natural frequency of trees has also been determined by measuring the power spectrum of tree velocity using a Laser Doppler Interferometer (Baker 1997). This method has been used to measure the natural frequency of building structures and provides results that are consistent with those obtained from free sway tests.

Models to predict natural frequency

In many cases the aim of mechanistic wind damage models is to predict the risk of damage to trees of different dimensions. Therefore, information on the natural frequency of trees with these dimensions is required. Because of the effort required to measure natural frequency, it is desirable to be able to predict the natural frequency of a tree from measurements of certain basic dimensions (e.g., height, diameter at breast height).

The same authors who collected data on natural frequencies usually also attempted to model these frequencies as a function of tree size. The theoretical basis for the equations used by these authors comes from either considering the tree as a beam with negligible mass and with a top load, or as a beam with distributed mass but without a top load.

Beam with a negligible mass and a top load

Sugden (1962) introduced an equation for predicting the period (T) of a weightless beam fixed at one end, with mass (M) concentrated on it at a distance (L) from the fixed end. This equation, as it appeared in the 1962 paper, did not include a term to account for the effect that the

Table 1 Summary of raw data from studies to determine tree natural frequency. Mean values are presented along with standard deviations () and ranges { }. (Authors *M73* Mayhead (1973a), *MGD75* Mayhead et al. (1975), *M91* Milne (1991), *G92* Gardiner (1992), *GBDW97* Gardiner et al. (1997), *FW99* Fleisch and Wilson (1999), *S62* Sugden (1962), *MM* Moore (unpublished data))

Genus/species	<i>N</i>	DBH (cm)	Height (m)	Stem mass (kg)	<i>f</i> (Hz)	Author
Spruce (<i>Picea</i> spp.)						
Sitka spruce	79	22.2 (3.2) {16.3, 30.0}	16.0 (2.2) {9.1, 18.9}	364.4 (125.5) {132.3, 701.5}	0.39 (0.08) {0.27, 0.71}	M73
Sitka spruce	69	17.6 (3.4) {7.2, 26.5}	12.4 (3.3) {6.0, 17.5}	–	0.58 (0.25) {0.30, 1.39}	MGD75
Sitka spruce	6	14.5 (3.0) {10.6, 18.3}	14.2 (0.9) {13.0, 15.5}	123.1 (53.4) {61.5, 190.8}	0.35 (0.06) {0.25, 0.40}	M91
Sitka spruce	10	16.7 (2.5) {13.0, 20.6}	15.1 (1.2) {13.4, 17.1}	167.3 (56.1) {74.8, 256.8}	0.33 (0.05) {0.26, 0.43}	G92
Sitka spruce	11	17.6 (3.1) {14.2, 22.6}	11.5 (0.7) {10.4, 12.7}	113.4 (28.2) {82.0, 167.0}	0.57 (0.12) {0.39, 0.74}	GBDW97
Norway spruce	8	21.1 (2.2) {17.0, 23.4}	15.3 (0.6) {14.6, 16.5}	323.0 (70.6) {233.5, 431.5}	0.39 (0.03) {0.34, 0.43}	M73
White spruce	6	21.0 (4.7) {17.0, 30.0}	13.8 (1.9) {12.1, 17.2}	–	0.40 (0.07) {0.32, 0.49}	FW99
Pine (<i>Pinus</i> spp.)						
Corsican pine	40	19.6 (3.2) {14.5, 27.4}	12.8 (1.6) {9.5, 15.9}	245.5 (121.6) {122.4, 610.2}	0.45 (0.10) {0.32, 0.83}	M73
Corsican pine	17	21.0 (5.3) {13.5, 28.0}	14.2 (2.3) {11.5, 17.7}	–	0.35 (0.04) {0.29, 0.42}	MGD75
Lodgepole pine	8	20.7 (2.3) {17.8, 24.4}	11.4 (1.0) {9.8, 12.5}	274.9 (70.4) {180.0, 347.4}	0.56 (0.09) {0.44, 0.65}	M73
Lodgepole pine	32	20.5 (3.7) {15.5, 30.0}	14.8 (1.8) {11.5, 18.5}	–	0.41 (0.07) {0.24, 0.60}	MGD75
Scots pine	16	24.0 (3.2) {19.5, 29.5}	16.1 (1.3) {13.3, 18.5}	–	0.35 (0.04) {0.29, 0.42}	MGD75
Red pine	284	16.7 (4.6) {6.9, 27.8}	13.6 (1.2) {7.9, 15.8}	–	0.32 (0.08) {0.16, 0.56}	S62
Douglas-fir						
Douglas-fir	8	25.9 (3.1) {21.8, 29.2}	17.8 (1.2) {16.5, 19.5}	612.5 (146.9) {390.0, 799.0}	0.33 (0.04) {0.30, 0.40}	M73
Douglas-fir	9	28.9 (7.4) {19.2, 38.1}	16.8 (2.1) {14.3, 20.0}	–	0.48 (0.10) {0.35, 0.66}	MM

diameter (*D*) of the rod has on its stiffness. The correct equation should have read:

$$T = \frac{b_0 \sqrt{ML^3}}{D^2} \quad (2)$$

Sugden (1962) noted that a tree could not be considered as a weightless beam because it had its mass distributed along its length. However, he did note that, in general, tree sway period decreased as diameter increased (height and branch weight remaining constant) and increased as height increased (all else remaining constant).

Later, Mayhead (1973a) fitted a number of models to the data collected during the 1960s by the British Forestry Commission. He found that sway period was best predicted using the following equation:

$$T = 0.86 + 0.74 \frac{H\sqrt{MH}}{\text{DBH}^2} \quad (3)$$

where, DBH = diameter at breast height (cm), and *H* = total tree height (m). The intercept term in Eq. 3 was significantly different from zero indicating a slight departure from theory. Mayhead (1973a) also noted the poor performance of the equation introduced by Sugden (1962). Unfortunately, this comment was based on the incorrect version of the equation. In fact, by substituting *L* = *H* and *D* = DBH into Eq. 2, it can be seen that the independent variables in Eq. 2 and Eq. 3 are identical.

Beam with distributed mass and without a top load

Gardiner (1992) used the Rayleigh method to develop an approximate theoretical relationship for the natural frequency of a tree by considering it as a beam with distributed mass. The Rayleigh method is based on the principle of conservation of energy; i.e., the energy in a freely vibrating system must remain constant if no

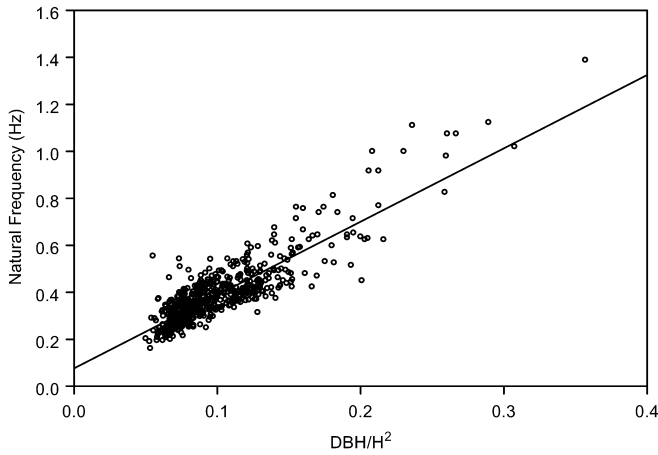


Fig. 1 Relationship between natural frequency and DBH/H^2 for all data listed in Table 1. The equation for the line passing through the data is given by Eq. 6

damping forces act to absorb it (Clough and Penzien 1993). By equating the maximum kinetic energy which occurs as the tree sways through the rest position to the change in gravitational potential energy which occurs between the rest condition and the limit of displacement, Gardiner (1992) showed that:

$$T = b_0 \frac{H^2}{d_L \sqrt{\frac{E}{\rho}}} \quad (4)$$

where d_L is basal diameter, E is Young's modulus of elasticity and ρ is wood density. If the ratio of Young's modulus to density is assumed to be approximately constant and basal diameter proportional to DBH, as was done by Gardiner (1992), then Eq. 4 can be simplified to:

$$T = b_0 \frac{H^2}{DBH} \text{ or } f = b_1 \frac{DBH}{H^2} \quad (5)$$

A plot of natural frequency versus DBH/H^2 for all data described in Table 1 is shown in Fig. 1. Data points were concentrated at values of DBH/H^2 less than 0.20 and some non-homogenous variance was apparent; therefore, the model was fitted to the data by weighted least squares (WLS) regression and model fit was assessed by Furnival's Index of Fit (Furnival 1961). The best model was obtained when the data were weighted by $(DBH/H^2)^{-1}$ with the resulting equation being:

$$f = 0.0766 + 3.1219 \frac{DBH}{H^2} \quad (6)$$

(0.0081) (0.0853)

where standard errors of the parameters are given in parentheses. This model was found to satisfy the assumptions of linear regression. The non-zero intercept in Eq. 6 indicates some departure from theory. While this intercept term is incompatible with the physics of tree oscillation, it needs to be retained in an empirically derived model to avoid under-prediction of f at low values of DBH/H^2 .

Species differences

Species differences were investigated by grouping trees according to genus (i.e., pines, spruces and Douglas-fir). Indicator variables were created for pines and Douglas-fir such that:

$$I_p = \begin{cases} 1 & \text{if genus} = \textit{Pinus} \\ 0 & \text{otherwise} \end{cases} \quad (7a)$$

$$I_d = \begin{cases} 1 & \text{if genus} = \textit{Pseudotsuga} \\ 0 & \text{otherwise} \end{cases} \quad (7b)$$

The model was refitted with spruce as the base genus and separate slopes and intercepts for pines and Douglas-fir, i.e.,

$$f = b_0 + b_1 I_p + b_2 I_d + b_3 \frac{DBH}{H^2} + b_4 I_p \frac{DBH}{H^2} + b_5 I_d \frac{DBH}{H^2} \quad (8)$$

where, $b_0 \dots b_5$ are model parameters. After accounting for DBH/H^2 , natural frequencies of pine were significantly lower than those of spruce and Douglas-fir ($P < 0.001$). The intercept terms for pine and Douglas-fir, and the slope term for Douglas-fir were not significantly different from those for spruce (i.e., b_1, b_2 and $b_4 = 0; P = 0.344$). The refitted model with parameter estimates and associated standard errors is given by Eq. 9:

$$f = 0.0948 + 3.4317 \frac{DBH}{H^2} - 0.7765 I_p \frac{DBH}{H^2} \quad (9)$$

(0.0068) (0.0725) (0.0459)

While he did not test for differences in natural frequency between species, Mayhead (1973b) suggested that if they did exist then they might be due to differences in factors such as branch form and flexibility, length of canopy and form factor. Sugden (1962) also noted that the heavier the crown of a tree, or the higher its center of mass, the lower the natural frequency. Therefore, for a given DBH/H^2 , pines could be expected to have higher centers of mass or heavier crowns, or both. For example, available allometric relationships indicate that ponderosa pine crowns carry more than twice the mass of Douglas-fir crowns on tree given height and diameter (Larsen and Hann 1987; Means et al. 1994). Additionally, in reducing Eq. 4 to Eq. 5, it was assumed that the ratio of E/ρ was constant. This assumption may not hold across species. Data from the USDA Wood Handbook (Forest Products Laboratory 1999) indicate that the ratio of E/ρ for green timber is approximately 15% greater for Douglas-fir, grand fir and Sitka spruce than for lodgepole pine and red pine.

Effect of branch removal

Milne (1991) and Gardiner (1992) found that oscillation frequencies of trees were greater after their branches had been removed. A plot of their data, collected from 16 Sitka spruce trees, along with those from 6 red pine (Sugden 1962) and 9 Douglas-fir trees (Moore, unpub-

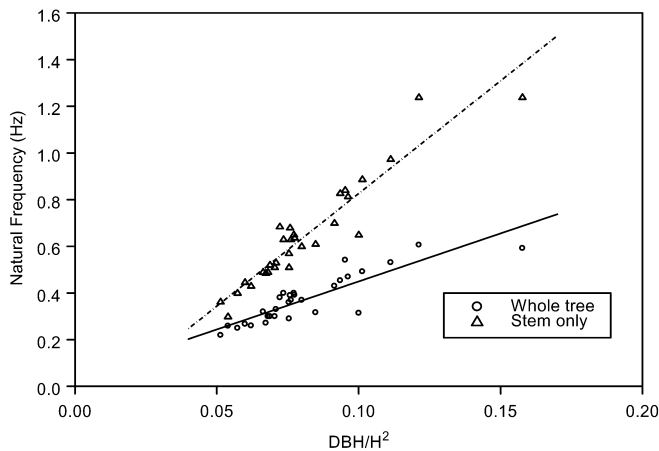


Fig. 2 Comparison of natural frequency between the stem only and the whole tree (stem + crown) for 16 Sitka spruce, 6 red pine and 9 Douglas-fir trees

lished data) is shown in Fig. 2. The difference in natural frequency between the branchless bole and the whole tree increases with increasing DBH/H^2 and can be attributed to a reduction in mass. For unpruned trees, a larger value of DBH/H^2 implies a greater crown length and crown mass. From Eq. 3 it can be seen that if crown mass increases with DBH/H^2 , this would act to attenuate the slope of the relationship between whole-tree natural frequency and DBH/H^2 . For those trees where data existed, significant linear relationships were found between branch mass and DBH/H^2 (Fig. 3). The increase in branch mass with increasing DBH/H^2 was much greater for the red pine trees measured by Sugden (1962) than for the Sitka spruce trees measured by Milne (1991) and Gardiner (1992). Data collected by Sugden (1962) came from a single stand where the relative variation in height was small compared to that in DBH.

Effect of snow loading

During the British Forestry Commission's aeromechanical experiments conducted at Rivo Forest in the 1980s (Papesch 1984), natural frequency measurements were made on two Sitka spruce trees ($H=9.25$ m) when 1–2 cm of snow was present in the crowns. A comparison of the results with those when no snow was present revealed an approximate 30% reduction in natural frequency. From Eq. 3 it is apparent that the additional mass of the snow will result in a reduction in natural frequency, however the actual mass of snow that accumulated in the crowns of the Rivo trees was not quantified.

Damping

The dynamic motion of a tree is attenuated by damping which acts to dissipate energy. This damping arises from a number of sources which Hoag et al. (1971) grouped

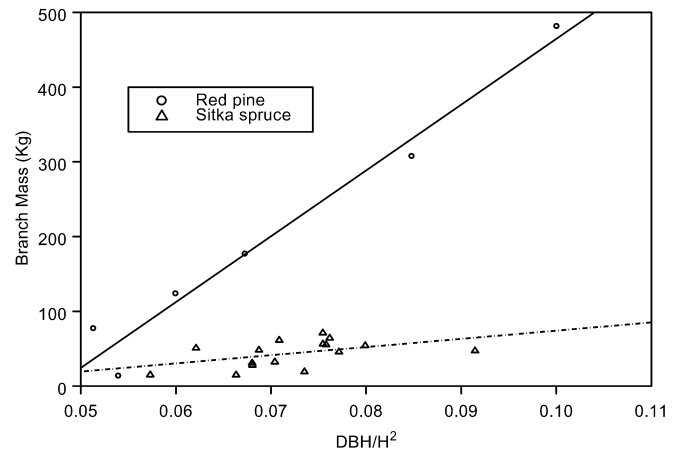


Fig. 3 Relationship between branch mass (kg) and DBH/H^2 for red pine and Sitka spruce trees

into internal and external. Internal damping is due to the friction of the root-soil connection (Mayhead et al. 1975), structural damping resulting from the movement of branches (Niklas 1992) and the internal friction of the wood (Milne 1992; Wood 1995). External damping is due to the aerodynamic drag of the crown and also to collisions between crowns of neighboring trees.

It is not usually possible to determine the amount of damping by using physical considerations because the basic energy-loss mechanisms are seldom fully understood (Clough and Penzien 1993). However, it is common practice to assume that damping is proportional to velocity (i.e., viscous damping) as this leads to a convenient form of the equations of motion. Assuming viscous damping, the total damping force (F_d) acting on a tree is given by:

$$F_d(t) = c\dot{y}(t) = 4\pi mf\xi\dot{y}(t) \quad (10)$$

where, c is the damping coefficient (kg s m^{-1}), f is the tree natural frequency (Hz), m is the mass (kg), ξ is the equivalent viscous damping ratio, and $\dot{y}(t)$ is the tree velocity at time t (m s^{-1}). The equivalent viscous damping ratio can be determined by experimental methods (Clough and Penzien 1993).

Measurement

A number of methods exist for calculating the viscous damping ratio of a tree. The most commonly used and simplest is the **free vibration decay method**. Using this method the damping ratio ξ (i.e., the proportion of critical damping) can be determined from the ratio of two peak displacements y_n and y_{n+m} measured over m cycles. From the equations of motion for damped free vibration, it can be shown that the ratio of these two successive peaks is given by:

$$\frac{y_n}{y_{n+m}} = \exp\left(\frac{2m\pi\xi}{\sqrt{1-\xi^2}}\right) \quad (11)$$

Taking the natural logarithms of both sides yields an expression for the logarithmic decrement of damping (δ):

$$\delta \equiv \ln \frac{y_n}{y_{n+m}} = \frac{2m\pi\xi}{\sqrt{1-\xi^2}} \quad (12)$$

For low values of damping (i.e., $\xi_{\text{approx}} < 0.2$), this equation can be solved for ξ to give:

$$\xi_{\text{approx}} = \frac{y_n - y_{n+m}}{2m\pi y_{n+m}} \quad (13)$$

Because of the simplifications made in solving Eq. 12, Eq. 13 consistently over predicts damping ratio, with the over prediction greatest for larger damping ratios. Fortunately, this over prediction occurs in a systematic manner and a correction factor can be applied. By solving Eq. 12 iteratively and comparing the results to Eq. 13 an empirical equation for correcting the results from Eq. 13 was developed.

$$\xi = \xi_{\text{approx}} \cdot \exp \left[-1.336(\xi_{\text{approx}})^{0.7175} \right] \quad (14)$$

A major advantage of the free vibration method is the minimal instrumentation requirements. If damping is truly of the linear viscous form, any set of m consecutive cycles will yield the same damping ratio. However, for many structures the damping ratio is often found to be amplitude dependent, with the damping ratio decreasing with decreasing amplitude of free vibration response

(Clough and Penzien 1993). For trees, Wood (1995) argued that the free vibration method does yield a constant value of the damping ratio.

Damping ratios can also be determined from methods that investigate the response of the tree to harmonic inputs of different frequencies. One of the most convenient of these is the **half-power (band-width) method**, whereby the damping ratio is determined from the frequencies at which the response amplitude is reduced to the level $1/\sqrt{2}$ times its peak value (Clough and Penzien 1993). These methods have not been used in previous studies on tree damping, but equipment developed by O'Sullivan and Ritchie (1992) and Rodgers et al. (1995) could be used to excite the trees at different frequencies. However, Rodgers et al. (1995) found that trees subjected to this type of forced loading eventually begin to oscillate in an elliptical pattern. In such situations, data would need to be ignored.

Previous studies

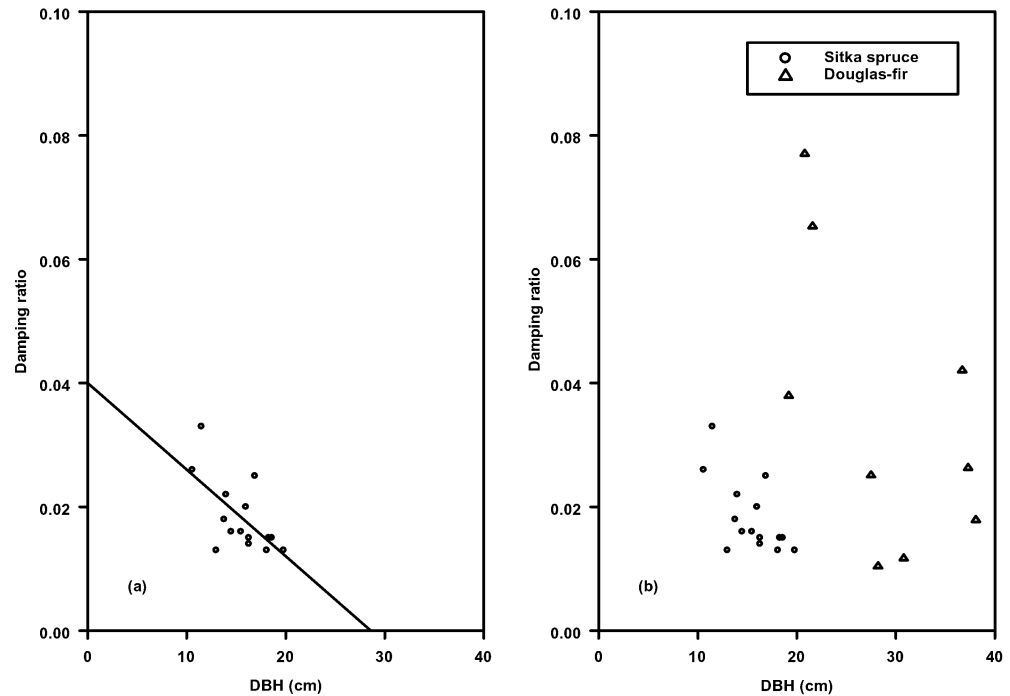
Many of the authors who measured the natural frequencies of trees also measured the viscous damping ratios. In one of the earliest studies, Loo (1975) measured the damping ratios of 18 Monterey pine (*Pinus radiata* D. Don) trees in New Zealand (Table 2). He investigated the effect of systematic pruning of the tree on the damping

Table 2 Summary of raw data from studies to determine the damping ratios of whole trees. Mean values are presented along with standard deviations () and ranges { }. (Authors: MGD75 Mayhead et al. (1975), BPM88 Blackburn et al. (1988), M91 Milne

(1991), G92 Gardiner (1992), GBDW97 Gardiner et al. (1997), FW99 Flesch and Wilson (1999), L75 Loo (1975), MM Moore (unpublished data)

Genus/species	n	DBH (cm)	Height (m)	Stem mass (kg)	ξ	Author
Spruce (<i>Picea</i> spp.)						
Sitka spruce	56	17.9 (3.2) {13.0, 26.5}	12.1 (3.3) {7.1, 17.5}	–	0.093 (0.023) {0.065, 0.140}	MGD75
Sitka spruce	3	18.6 (3.8) {14.3, 21.4}	13.2 (0.5) {12.7, 13.6}	167.3 (66.9) {92.0, 220.0}	0.044 (0.015) {0.027, 0.056}	BPM88
Sitka spruce	6	14.5 (3.0) {10.6, 18.3}	14.2 (0.9) {13.0, 15.5}	123.1 (53.4) {61.5, 190.8}	0.123 (0.033) {0.099, 0.189}	M91
Sitka spruce	10	16.7 (2.5) {13.0, 20.6}	15.1 (1.2) {13.4, 17.1}	167.3 (56.1) {74.8, 256.8}	0.068 (0.012) {0.047, 0.076}	G92
Sitka spruce	11	17.6 (3.1) {14.2, 22.6}	11.5 (0.7) {10.4, 12.7}	113.4 (28.2) {82.0, 167.0}	0.122 (0.054) {0.060, 0.250}	GBDW97
White spruce	6	21.0 (4.7) {17.0, 30.0}	13.8 (1.9) {12.1, 17.2}	–	0.115 (0.037) {0.072, 0.162}	FW99
Pine (<i>Pinus</i> spp.)						
Monterey pine	18	24.0 (3.9) {17.6, 28.6}	10.9 (1.7) {7.8, 13.2}	–	0.060 (0.011) {0.038, 0.075}	L75
Lodgepole pine	32	20.5 (3.7) {15.5, 30.0}	14.8 (1.8) {11.5, 18.5}	–	0.065 (0.014) {0.042, 0.099}	MGD75
Scots pine	16	24.0 (3.2) {19.5, 29.5}	16.1 (1.3) {13.3, 18.5}	–	0.060 (0.007) {0.047, 0.071}	MGD75
Douglas-fir						
Douglas-fir	9	28.9 (7.4) {19.2, 38.1}	16.8 (2.1) {14.3, 20.0}	–	0.154 (0.061) {0.052, 0.227}	MM

Fig. 4 Relationship between damping ratio and DBH for **a** Sitka spruce, and **b** Sitka spruce and Douglas-fir



ratio but these data were not presented. Mayhead et al. (1975) also investigated the effect of systematic pruning and found that around half the silhouette area of the crown had to be removed before the damping ratio was greatly affected. Removal of the last (uppermost) section of the crown had the largest effect on damping. Their experiments with adding a mass, equivalent to that of the crown, to the top of a branchless bole also demonstrated that damping was due to the aerodynamic drag of the canopy rather than its mass. Milne (1991) measured the damping ratios on six Sitka spruce trees and found that, for the tree size and stand density conditions he worked in, crown clashing with neighboring trees and aerodynamic drag on foliage accounted for 50 and 40% of total damping, respectively. Internal damping accounted for the remaining 10%. The amount of damping due to crown clashing depended on the distance to neighbors, as well as on the sizes of the chosen tree and its neighbors.

Modeling

Few, if any, studies have attempted to model damping due to the lack of understanding of the energy-loss mechanisms in trees. Milne (1991) found that for six Sitka spruce trees, internal damping was inversely related to stem diameter (Fig. 4a). This linear relationship appeared to hold when additional Sitka spruce data were included ($r^2=0.40$, $P=0.011$) but not when data from tests on de-branched Douglas-fir trees were included ($P=0.489$; Fig. 4b).

Much work was done by the British Forestry Commission in the 1960s and 1970s to determine the aerodynamic drag of different species (Raymer 1962;

Table 3 Mean estimates of the external damping coefficient (c_E) and the power of wind velocity (n). Standard errors of the means are given in parentheses

Species	c_E ($N s m^{-1}$) ^{1/n}	n
Sitka spruce	18.330 (1.405)	1.416 (0.107)
Corsican pine	15.978 (1.480)	1.383 (0.150)
Lodgepole pine	24.171 (–)	1.297 (–)
Scots pine	15.751 (0.978)	1.377 (0.198)
Douglas-fir	23.467 (1.293)	1.247 (0.103)
Western hemlock	13.065 (0.578)	1.253 (0.122)
Grand fir	24.624 (–)	1.247 (–)

Mayhead 1973b; Mayhead et al. 1975). Using data from wind-tunnel tests on full-scale trees (DBH =10–15 cm, Height =6–8 m), these authors computed drag coefficients for the species tested. From the same data it was possible to calculate the external damping force (F_{DE}) resulting from the aerodynamic drag caused by a tree moving through the air. The aerodynamic damping force is given by:

$$F_{DE} = c_E U^n \quad (15)$$

where, c_E is the external damping coefficient ($N s m^{-1}$)^{1/n}, U is wind velocity and n is the power of wind velocity (Hoag et al. 1971). It should be noted here that for $n \neq 1$, the assumption of viscous damping is violated. Using the data from Mayhead (1973b), relationships of the form given in Eq. 15 were developed between the drag force and wind speed for a range of species. Values of c_E and n were estimated for each species using non-linear least squares regression and are presented in Table 3. The values of c_E for Sitka spruce were similar to the viscous damping coefficients obtained by Gardiner et al. (1997).

However, the value of n was different from 1 for all species, indicating that aerodynamic damping was not a linear function of velocity. For non-streamlined rigid bodies, aerodynamic considerations lead to the drag force being proportional to the second power of velocity (i.e., $n=2$). However, for streamlined rigid bodies, the exponent on the velocity term is less than 2.0 (Vogel 1988). In trees, streamlining of the branches can lead to a reduction in crown frontal area with increasing wind velocity (Hoag et al. 1971). The values of n could, therefore, be used to examine the relative streamlining ability of different tree species. Unfortunately, this was not possible for the data collected by Mayhead (1973b) because of the small number of trees tested (only one tree was measured for some species).

The other major component of external damping is contact between crowns of adjacent trees. Several authors (Milne 1991; Gardiner 1992; Gardiner et al. 1997) have measured the damping ratios of trees both when the crown of the test tree was in contact with those of its neighbors, and when crown contact was prevented. Milne (1991) used a neighbor interference parameter, defined as the average ratio of D_i/l_i^2 , where D and l are the diameter of, and distance to neighbor i , to account for the effect of neighboring trees on damping. However, the data set used by Milne (1991) was collected in a stand containing a limited range of tree sizes. Therefore, measurements of damping ratios would need to be made on trees of different sizes growing under a wider range of stand densities to confirm Milne's (1991) assumption. Rather than using the ratio of D_i/l_i^2 , another possible indicator of the degree of crown contact is stand density index (SDI). This is a common measure of site occupancy used by forest managers, particularly in the western USA, and is defined as (Reineke 1933):

$$SDI = TPH \left(\frac{QMD}{25.4} \right)^{1.605} \quad (16)$$

where, TPH is trees per hectare, and QMD is quadratic mean diameter (cm). SDI accounts for both the size of and the average area occupied by trees within a stand. A comparison of Eq. 16 with the neighbor interference parameter used by Milne (1991) reveals that the two are quite similar, particularly if SDI is calculated for the region of the stand occupied by the tree of interest and its neighbors.

Conclusions

Based on a review and analysis of previous studies that have measured tree sway frequencies and damping ratios, the following conclusions can be drawn.

1. Natural frequency of whole trees can be reliably estimated from tree height and DBH, information that is commonly available in stand record tables.

2. For a given tree size, pines had lower natural frequencies than both spruces and Douglas-fir. Some variation in species is driven by either or both of two factors: (1) differences in crown weight for a given DBH and H ; and (2) differences in the ratio of Young's modulus of elasticity to wood density.
3. The natural frequency of trees with their branches removed was higher than for those with branches, the difference increasing with increasing DBH/ H^2 . This increase is possibly due to proportionally larger crown mass in trees with a larger ratio of DBH/ H^2 as well as increased aerodynamic damping because of a higher velocity of oscillation.
4. Internal damping ratios are generally less than 0.05 and do not appear to be related to tree size.
5. External damping is mainly due to crowns of adjacent trees clashing together and aerodynamic drag on the foliage, with the latter being a nonlinear function of velocity.

The existing information on the dynamic properties of trees that has been compiled and summarized here is an important source of knowledge for researchers investigating the dynamic behavior of trees subjected to wind loading. It is hoped that in the future the data sets compiled here can be expanded to include results from any previously unknown studies.

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